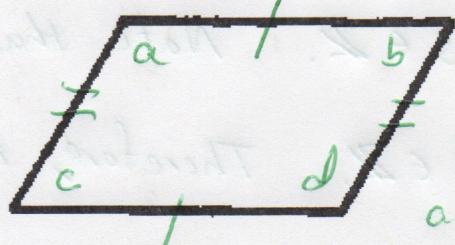


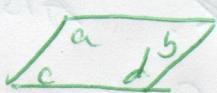
Full credit will only be given to answers that contain full solutions with work shown/explanations.  
There are two questions worth a total of 10 points.

1. (3 pts) Describe all symmetries of a parallelogram that is not a rectangle or a rhombus. Create the corresponding Cayley table.

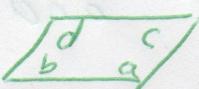


Note that any symmetry transformation must preserve the locations of the obtuse angles (corners a, b) and the locations of congruent sides (sides ab, cd / sides ac, bd).

Therefore, there are only 2 orientations:



$$R_0 = \text{Id}$$



$$R_{180^\circ}$$

Cayley table:

	<u>Id</u>	<u><math>R_{180^\circ}</math></u>
<u>Id</u>	Id	$R_{180^\circ}$
$R_{180^\circ}$	$R_{180^\circ}$	Id.

2. (3 pts) Define a group.

See textbook.

(a) (4 pts) Prove that the set  $S = \{2^r \mid r \in \mathbb{Z}\}$  is a group under multiplication.

First we check that the set is closed under the operation. Let  $x, y \in S$ . Then  $x = 2^r, y = 2^s$  for some  $r, s \in \mathbb{Z}$ . Note that  $x \cdot y = 2^r \cdot 2^s = 2^{r+s}$  and  $r+s \in \mathbb{Z}$ . Therefore, the set is closed.

Now we check for an identity. Note that  $1 \in S$  since  $1 = 2^0$  and  $0 \in \mathbb{Z}$ . Also,  $1 \cdot x = x = x \cdot 1$  for any  $x \in S$ .

For inverses, we observe that for  $x = 2^r \in S$ , the element number  $y = 2^{-r} \in S$  (since  $-r \in \mathbb{Z}$  if  $r \in \mathbb{Z}$ ) and  $xy = 2^r \cdot 2^{-r} = 1 = yx$ . Therefore,  $y$  is an inverse for  $x$ .

Lastly, we note that multiplication of the real numbers is associative in general. Since  $S$  is a subset of the reals, multiplication is associative on this set as well.

(We could also use the associativity of addition for the integers, since  $(xy)z = (2^r \cdot 2^s) \cdot 2^t = 2^{r+s} \cdot 2^t = 2^{(r+s)+t}$ .)