

Full credit will only be given to answers that contain full solutions with work shown/explanations.
There are three questions worth a total of 10 points.

1. (3 pts) Define *group order* and *element order*. Briefly explain how the two ideas are related if G is a cyclic group.

See text book for definitions.

There are a variety of ways that element order and group order are related. One significant way is that if $G = \langle a \rangle$, then $|G| = |a|$.

2. (3 pts) Suppose G is a cyclic group generated by an element $a \in G$ with $|a| = 15$. List all generators of G and justify your answer.

Using Corollary 3 of Theorem 4.2, we know that a^j is a generator for G if and only if $\gcd(n, j) = 1$.

Therefore, the generators of G are

$$a, a^2, a^4, a^7, a^8, a^{11}, a^{13}, a^{14}.$$

3. (4 pts) Let G be a group and let a be a fixed element in G . Recall that the *centralizer* of a in G is

$$C(a) = \{g \in G \mid ga = ag\}.$$

Show that $C(a)$ is a subgroup of G .

To prove this, I'll use the two-step subgroup test.

First, note that $e \in C(a)$. This implies that $e \in C(a)$ and so $C(a)$ is nonempty.

Now let $g \in C(a)$. I want to show $g^{-1} \in C(a)$.

We know $ag = ga$

$$g^{-1}(ag)g^{-1} = g^{-1}(ga)g^{-1}$$

$$g^{-1}a(g^{-1}g) = (g^{-1}g)ag^{-1}$$

$$\begin{aligned} & ag \\ & g^{-1}a e = e a g^{-1} \\ & g^{-1}a = ag^{-1} \end{aligned}$$

Therefore $g^{-1} \in C(a)$.

Now let $g, h \in C(a)$. To show $gh \in C(a)$. We first note that $ga = ag$ and $ha = ah$. Then,

$$\begin{aligned} (gh)a &= g(ha) = g(ah) \\ &= (ga)h = (ag)h = a(gh). \end{aligned}$$

Therefore $gh \in C(a)$.